10.5

Factoring $x^2 + bx + c$

What you should learn

GOAL 1 Factor a quadratic expression of the form $x^2 + bx + c$.

GOAL 2 Solve quadratic equations by factoring.

Why you should learn it

▼ To help solve **real-life** problems, such as finding the width of a stone border in **Example 7**.

GOAL 1 FACTORING A QUADRATIC TRINOMIAL

To **factor** a quadratic expression means to write it as the product of two linear expressions. In this lesson, you will learn how to factor quadratic trinomials that have a leading coefficient of 1.

FACTORING $x^2 + bx + c$

You know from the FOIL method that $(x + p)(x + q) = x^2 + (p + q)x + pq$. So to factor $x^2 + bx + c$, you need to find numbers p and q such that

$$p + q = b \quad \text{and} \quad pq = c$$

because $x^2 + (p + q)x + pq = x^2 + \mathbf{b}x + \mathbf{c}$ if and only if $p + q = \mathbf{b}$ and $pq = \mathbf{c}$.

Example: $x^2 + \mathbf{6}x + \mathbf{8} = (x + 4)(x + 2)$ $\quad 4 + 2 = \mathbf{6}$ and $4 \cdot 2 = \mathbf{8}$

EXAMPLE 1 *Factoring when b and c are Positive*

Factor $x^2 + 3x + 2$.

SOLUTION

For this trinomial, $b = 3$ and $c = 2$. You need to find two numbers whose sum is 3 and whose product is 2.

$x^2 + 3x + 2 = (x + p)(x + q)$ **Find p and q when $p + q = 3$ and $pq = 2$.**

$= (x + 1)(x + 2)$ **$p = 1$ and $q = 2$**

✓**CHECK:** You can check the result by multiplying.

$(x + 1)(x + 2) = x^2 + 2x + x + 2$ **Use FOIL to check.**

$= x^2 + 3x + 2$ **Simplify.**

EXAMPLE 2 *Factoring when b is Negative and c is Positive*

Factor $x^2 - 5x + 6$.

SOLUTION

Because b is negative and c is positive, both p and q must be negative numbers. Find two numbers whose sum is -5 and whose product is 6.

$x^2 - 5x + 6 = (x + p)(x + q)$ **Find p and q when $p + q = -5$ and $pq = 6$.**

$= (x - 2)(x - 3)$ **$p = -2$ and $q = -3$**

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 3 Factoring when b and c are Negative

Factor $x^2 - 2x - 8$.

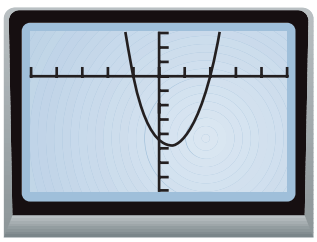
SOLUTION For this trinomial, $b = -2$ and $c = -8$. Because c is negative, you know that p and q cannot both have negative values.

$$x^2 - 2x - 8 = (x + p)(x + q) \qquad \text{Find } p \text{ and } q \text{ when } p + q = -2 \text{ and } pq = -8.$$

$$= (x + 2)(x - 4) \qquad p = 2 \text{ and } q = -4$$

✓ **CHECK:** Use a graphing calculator. Graph $y = x^2 - 2x - 8$ and $y = (x + 2)(x - 4)$ on the same screen.

The graphs coincide, so your answer is correct.



EXAMPLE 4 Factoring when b is Positive and c is Negative

Factor $x^2 + 7x - 18$.

SOLUTION For this trinomial, $b = 7$ and $c = -18$. Because c is negative, you know that p and q cannot both have negative values.

$$x^2 + 7x - 18 = (x + p)(x + q) \qquad \text{Find } p \text{ and } q \text{ when } p + q = 7 \text{ and } pq = -18.$$

$$= (x + 9)(x - 2) \qquad p = 9 \text{ and } q = -2$$

. . . . . .

STUDENT HELP

Look Back
For help with finding the discriminant, see p. 541.

It is important to realize that *many* quadratic trinomials with integer coefficients cannot be factored into linear factors with integer coefficients. A quadratic trinomial $x^2 + bx + c$ can be factored (using integer coefficients) only if the *discriminant* is a perfect square.

EXAMPLE 5 Using the Discriminant

Tell whether the trinomial can be factored.

a. $x^2 + 3x - 4$ **b.** $x^2 + 3x - 6$

SOLUTION Find the discriminant.

a. $b^2 - 4ac = 3^2 - 4(1)(-4) \qquad a = 1, b = 3, \text{ and } c = -4$

$$= 25 \qquad \text{Simplify.}$$

▶ The discriminant is a perfect square, so the trinomial can be factored.

b. $b^2 - 4ac = 3^2 - 4(1)(-6) \qquad a = 1, b = 3, \text{ and } c = -6$

$$= 33 \qquad \text{Simplify.}$$

▶ The discriminant is not a perfect square, so the trinomial cannot be factored.

GOAL 2 SOLVING QUADRATIC EQUATIONS BY FACTORING

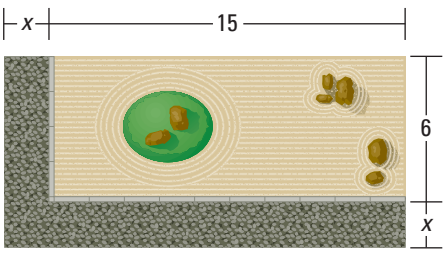
EXAMPLE 6 *Solving a Quadratic Equation*

$x^2 - 3x = 10$	**Write equation.**
$x^2 - 3x - 10 = 0$	**Write in standard form.**
$(x - 5)(x + 2) = 0$	**Factor left side.**
$(x - 5) = 0$ or $(x + 2) = 0$	**Use zero-product property.**
$x - 5 = 0$	**Set first factor equal to 0.**
$x = 5$	**Solve for *x*.**
$x + 2 = 0$	**Set second factor equal to 0.**
$x = -2$	**Solve for *x*.**

▶ The solutions are 5 and -2. Check these in the original equation.

EXAMPLE 7 *Writing a Quadratic Model*

LANDSCAPE DESIGN You are putting a stone border along two sides of a rectangular Japanese garden that measures 6 yards by 15 yards. Your budget limits you to only enough stone to cover 46 square yards. How wide should the border be?



SOLUTION

Begin by drawing and labeling a diagram.

$$\boxed{\text{Area of border}} = \boxed{\text{Total area}} - \boxed{\text{Garden area}}$$

$46 = (x + 15)(x + 6) - (15)(6)$	**Write quadratic model.**
$46 = x^2 + 21x + 90 - 90$	**Multiply.**
$0 = x^2 + 21x - 46$	**Write in standard form.**
$0 = (x + 23)(x - 2)$	**Factor.**
$(x + 23) = 0$ or $(x - 2) = 0$	**Use zero-product property.**
$x + 23 = 0$	**Set first factor equal to 0.**
$x = -23$	**Solve for *x*.**
$x - 2 = 0$	**Set second factor equal to 0.**
$x = 2$	**Solve for *x*.**

▶ The solutions are -23 and 2. Only $x = 2$ is a reasonable solution, because negative values for dimension do not make sense. Make the border 2 yards wide.

FOCUS ON CAREERS

REAL LIFE LANDSCAPE DESIGNERS plan and map out the appearance of outdoor spaces like parks, gardens, golf courses, and other recreation areas.

INTERNET CAREER LINK
www.mcdougallittell.com

GUIDED PRACTICE

Vocabulary Check ✓

1. What does it mean to factor a quadratic expression?

Concept Check ✓

2. Factor $x^2 - 4x + 3$. When testing possible factorizations, why is it unnecessary to test $(x - 1)(x + 3)$ and $(x + 1)(x - 3)$?

3. Factor $x^2 + 2x - 3$. When testing possible factorizations, why is it unnecessary to test $(x - 1)(x - 3)$ and $(x + 1)(x + 3)$?

4. Can a trinomial $x^2 + bx + c$, where b and c are integers, be factored with integer coefficients if its discriminant is 35? Explain.

Skill Check ✓

Match the trinomial with a correct factorization.

5. $x^2 - x - 20$
6. $x^2 + x - 20$
7. $x^2 + 9x + 20$
8. $x^2 - 9x + 20$

A. $(x + 5)(x - 4)$
B. $(x + 4)(x + 5)$
C. $(x - 4)(x - 5)$
D. $(x + 4)(x - 5)$

Use the discriminant to decide whether the equation can be solved by factoring. Explain your reasoning.

9. $x^2 - 4x + 4 = 0$
10. $x^2 - 4x - 5 = 0$
11. $x^2 - 4x - 6 = 0$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 806.

FACTORED FORM **Choose the correct factorization. If neither is correct, find the correct factorization.**

12. $x^2 + 7x + 12$
    A. $(x + 6)(x + 2)$
    B. $(x + 4)(x + 3)$
13. $x^2 - 10x + 16$
    A. $(x - 4)(x - 4)$
    B. $(x - 8)(x - 2)$
14. $x^2 + 11x - 26$
    A. $(x - 13)(x + 2)$
    B. $(x - 13)(x - 2)$

FACTORING TRINOMIALS **Factor the trinomial.**

15. $x^2 + 8x - 9$
16. $t^2 - 10t + 21$
17. $b^2 + 5b - 24$
18. $w^2 + 13w + 36$
19. $y^2 - 3y - 18$
20. $c^2 + 14c + 40$
21. $m^2 - 7m - 30$
22. $32 + 12n + n^2$
23. $44 - 15s + s^2$
24. $z^2 + 65z + 1000$
25. $x^2 - 45x + 450$
26. $d^2 - 33d - 280$

STUDENT HELP

HOMEWORK HELP

Examples 1–4: Exs. 12–26, 48–51
Example 5: Exs. 42–47
Example 6: Exs. 27–41
Example 7: Exs. 56–60

SOLVING QUADRATIC EQUATIONS **Solve the equation by factoring.**

27. $x^2 + 7x + 10 = 0$
28. $x^2 + 5x - 14 = 0$
29. $x^2 - 9x = -14$
30. $x^2 + 32x = -220$
31. $x^2 + 16x = -15$
32. $x^2 + 3x = 54$
33. $x^2 + 8x = 65$
34. $-x + x^2 = 56$
35. $x^2 - 20x = -51$
36. $x^2 - 5x = 84$
37. $x^2 + 3x - 31 = -3$
38. $x^2 - 2x - 19 = -4$
39. $x^2 - x - 8 = 82$
40. $x^2 + 42 = 13x$
41. $x^2 - 9x + 18 = 2x$

USING THE DISCRIMINANT **Tell whether the quadratic expression can be factored with integer coefficients. If it can, find the factors.**

42. $t^2 + 7t - 144$ **43.** $y^2 + 19y + 60$ **44.** $x^2 - 11x + 24$

45. $w^2 - 6w + 16$ **46.** $z^2 - 26z - 87$ **47.** $b^2 + 14b + 35$

CHECKING GRAPHICALLY **Solve the equation by factoring. Then use a graphing calculator to check your answer.**

48. $x^2 - 17x + 30 = 0$ **49.** $x^2 + 8x = 105$

50. $x^2 - 20x + 21 = 2$ **51.** $x^2 + 52x + 680 = 40$

WRITING EQUATIONS **Write a quadratic equation with the given solutions.**

52. 12 and -21 **53.** -5 and -6 **54.** -41 and 5 **55.** 427 and 0

GEOMETRY CONNECTION **In Exercises 56–58, consider a rectangle having one side of length $x - 6$ and having an area given by $A = x^2 - 17x + 66$.**

56. Use factoring to find an expression for the other side of the rectangle.

57. If the area of the rectangle is 84 square feet, what are possible values of x?

58. For the value of x found in Exercise 57, what are the dimensions of the rectangle?

STUDENT HELP

TABLE OF FORMULAS

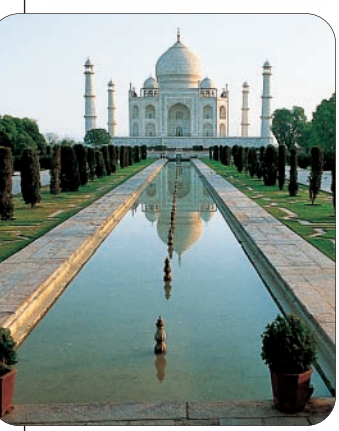
For help with area, see the Table of Formulas, p. 813.

GEOMETRY CONNECTION **Consider a circle whose radius is greater than 9 and whose area is given by $A = \pi(x^2 - 18x + 81)$. (Use $\pi \approx 3.14$.)**

59. Use factoring to find an expression for the radius of the circle.

60. If the area of the circle is 12.56 square meters, what is the value of x?

FOCUS ON APPLICATIONS

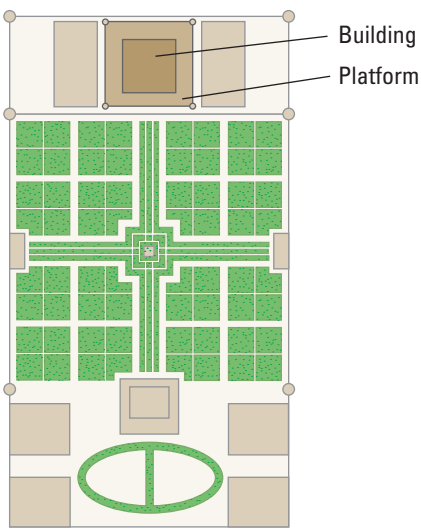


MAKING A SIGN **In Exercises 61 and 62, a triangular sign has a base that is 2 feet less than twice its height. A local zoning ordinance restricts the surface area of street signs to no more than 20 square feet.**

61. Write an inequality involving the height of the triangle that represents the largest triangular sign allowed.

62. Find the base and height of the largest triangular sign that meets the zoning ordinance.

THE TAJ MAHAL **In Exercises 63 and 64, refer to the illustration at the right of the Taj Mahal.**



63. The platform is about 38 meters wider than the main building. The total area of the platform is about 9025 square meters. Find the dimensions of the platform and the base of the building. (Assume each is a square.)

64. The entire complex of the Taj Mahal is about 245 meters longer than it is wide. The area of the entire complex is about 167,750 square meters. What are the dimensions of the entire complex? Explain your steps in finding the solution.

REAL LIFE

TAJ MAHAL

It took more than 20,000 daily workers 22 years to complete the Taj Mahal around 1643 in India. Constructed primarily of white marble and red sandstone, the Taj Mahal is renowned for its beauty.

INTERNET APPLICATION LINK

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Test Preparation

65. MULTIPLE CHOICE The length of a rectangular plot of land with an area of 880 square meters is 24 meters more than its width. A paved area measuring 8 meters by 12 meters is placed on the plot. If w represents the width of the plot of land in meters, which of the following equations can be factored to find possible values of the width of the land?

(A) $w^2 + 24w = 880$ (B) $w^2 - 24w = 880$

(C) $w^2 + 24w = -880$ (D) $w^2 - 24w = -880$

66. MULTIPLE CHOICE A triangle's base is 16 feet less than 2 times its height. If h represents the height in feet, and the total area of the triangle is 48 square feet, which of the following equations can be used to determine the height?

(A) $2h + 2(h + 4) = 48$ (B) $h^2 - 8h = 48$

(C) $h^2 + 8h = 48$ (D) $2h^2 - 16h = 48$

67. MULTIPLE CHOICE Which of the following equations does *not* have solutions that are integers?

(A) $x^2 + 21x + 100 = -10$ (B) $x^2 - 169 = 0$

(C) $x^2 - 8x - 105 = 0$ (D) $x^2 - 15x - 75 = 0$

Challenge

FACTORING CHALLENGE **In Exercises 68–71, n is a positive integer. Factor the expression. (*Hint*: $(a^n)^2 = a^{2n}$)**

EXTRA CHALLENGE
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68. $a^{2n} - b^{2n}$ **69.** $a^{2n} + 2a^nb^n + b^{2n}$

70. $a^{2n} + 18a^nb^n + 81b^{2n}$ **71.** $5a^{2n} - 9a^nb^n - 2b^{2n}$

MIXED REVIEW

FINDING THE GCF **Find the greatest common factor.** (Skills Review, p. 777)

72. 30, 45 **73.** 49, 64 **74.** 412, 18

75. 77, 91 **76.** 20, 32, 40 **77.** 36, 54, 162

MULTIPLYING EXPRESSIONS **Find the product.** (Review 10.2, 10.3)

78. $3q(q^3 - 5q^2 + 6)$ **79.** $(y + 9)(y - 4)$ **80.** $(7x - 11)^2$

81. $(5 - w)(12 + 3w)$ **82.** $(3a - 2)(4a + 6)$ **83.** $(2b - 4)(b^3 + 4b^2 + 5b)$

84. $(9x + 8)(9x - 8)$ **85.** $\left(6z + \frac{1}{3}\right)^2$ **86.** $(5t - 3)(4t - 10)$

SOLVING FACTORED EQUATIONS **Solve the equation.** (Review 10.4)

87. $(x + 12)(x + 7) = 0$ **88.** $(z + 2)(z + 3) = 0$ **89.** $(t - 19)^2 = 0$

90. $\left(b - \frac{2}{5}\right)\left(b - \frac{5}{6}\right) = 0$ **91.** $(x - 9)(x - 6) = 0$ **92.** $(y + 47)(y - 27) = 0$

93. $(z - 1)(4z + 2) = 0$ **94.** $(3a - 8)(a + 5) = 0$ **95.** $(4n - 6)^3 = 0$

96. DISASTER RELIEF You drop a box of supplies from a helicopter at an altitude of 40 feet above a drop area. Use a vertical motion model to find the time it takes the box to reach the ground. (Review 9.5 for 10.6)