10.2 Multiplying Polynomials

What you should learn

GOAL 1 Multiply two polynomials.

GOAL 2 Use polynomial multiplication in **real-life** situations, such as calculating the area of a window in **Example 5**.

Why you should learn it

▼ To solve **real-life** problems such as finding a model for movie ticket revenue in **Exs. 58 and 59**.

GOAL 1 MULTIPLYING POLYNOMIALS

In Lesson 2.6 you learned how to multiply a polynomial by a monomial by using the distributive property.

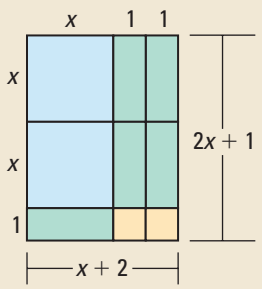
$$(3x)(2x^2 - 5x + 3) = (3x)(2x^2) + (3x)(-5x) + (3x)(3) = 6x^3 - 15x^2 + 9x$$

In the following activity, you will see how an area model illustrates the multiplication of two binomials.

ACTIVITY
Developing Concepts

Investigating Binomial Multiplication

The rectangle shown at the right has a width of $(x + 2)$ and a height of $(2x + 1)$.



1. Copy the model. What is the area of each part of the rectangle?
2. Find the product of $(x + 2)$ and $(2x + 1)$ by adding the areas of the parts to get an expression for total area.
3. Copy and complete the equation: $(x + 2)(2x + 1) = \underline{?}$.
4. Use an area model to multiply $(x + 3)$ and $(2x + 4)$.

Another way to multiply two binomials is to use the distributive property *twice*.

First use $(a + b)(c + d) = a(c + d) + b(c + d)$.

Then use $a(c + d) = ac + ad$ and $b(c + d) = bc + bd$.

This shows that $(a + b)(c + d) = ac + ad + bc + bd$.

This property can also be applied to binomials of the form $a - b$ or $c - d$.

EXAMPLE 1 Using the Distributive Property

Find the product $(x + 2)(x - 3)$.

STUDENT HELP
Look Back
For help with the distributive property, see p. 100.

SOLUTION

$$(x + 2)(x - 3) = x(x - 3) + 2(x - 3) \qquad (b + c)a = ba + ca$$
$$= x^2 - 3x + 2x - 6 \qquad a(b - c) = ab - ac$$
$$= x^2 - x - 6 \qquad \text{Combine like terms.}$$

When you multiply two binomials, you can remember the results given by the distributive property by means of the **FOIL** pattern. Multiply the **F**irst, **O**uter, **I**nner, and **L**ast terms.

Product of First terms | Product of Outer terms | Product of Inner terms | Product of Last terms

$$(3x + 4)(x + 5) = 3x^2 + 15x + 4x + 20$$
$$= 3x^2 + 19x + 20$$

STUDENT HELP

INTERNET HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 2 Multiplying Binomials Using the FOIL Pattern

$$(3x - 4)(2x + 1) = \underset{\text{F}}{6x^2} + \underset{\text{O}}{3x} - \underset{\text{I}}{8x} - \underset{\text{L}}{4}$$ **Mental math**

$$= 6x^2 - 5x - 4$$ **Simplify.**

. . . . . .

To multiply two polynomials that have three or more terms, remember that *each term of one polynomial must be multiplied by each term of the other polynomial.* Use a vertical or a horizontal format. Write each polynomial in standard form.

EXAMPLE 3 Multiplying Polynomials Vertically

Find the product $(x - 2)(5 + 3x - x^2)$.

SOLUTION Align like terms in columns.

$$\begin{array}{rl} -x^2 + 3x + 5 & \text{Standard form} \\ \times \quad x - 2 & \text{Standard form} \\ \hline 2x^2 - 6x - 10 & \leftarrow -2(-x^2 + 3x + 5) \\ -x^3 + 3x^2 + 5x & \leftarrow x(-x^2 + 3x + 5) \\ \hline -x^3 + 5x^2 - x - 10 & \text{Combine like terms.} \end{array}$$

EXAMPLE 4 Multiplying Polynomials Horizontally

Find the product $(4x^2 - 3x - 1)(2x - 5)$.

STUDENT HELP

Look Back
For help with multiplying exponential expressions, see p. 451.

SOLUTION Multiply $2x - 5$ by each term of $4x^2 - 3x - 1$.

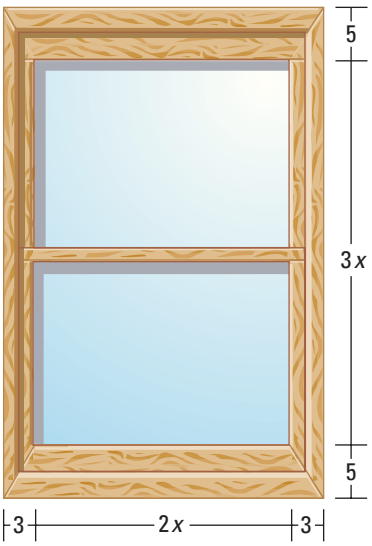
$$(4x^2 - 3x - 1)(2x - 5) = 4x^2(2x - 5) - 3x(2x - 5) - 1(2x - 5)$$
$$= 8x^3 - 20x^2 - 6x^2 + 15x - 2x + 5$$
$$= 8x^3 - 26x^2 + 13x + 5$$

GOAL 2 USING POLYNOMIALS IN REAL LIFE

REAL LIFE Carpentry

EXAMPLE 5 *Multiplying Binomials to Find an Area*

The diagram at the right shows the basic dimensions for a window. The glass portion of the window has a height-to-width ratio of 3 : 2. The framework adds 6 inches to the width and 10 inches to the height.



a. Write a polynomial expression that represents the total area of the window, including the framework.

b. Find the area when $x = 10$, 11, 12, 13, and 14.

SOLUTION

a. Use a verbal model.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

$$\text{Total area} = \text{Height of window} \cdot \text{Width of window}$$

LABELS

Total area $= A$ (square inches)

Height of window $= 3x + 10$ (inches)

Width of window $= 2x + 6$ (inches)

ALGEBRAIC MODEL

$$A = (3x + 10) \cdot (2x + 6) \quad \text{Area model}$$

$$= 6x^2 + 18x + 20x + 60 \quad \text{FOIL pattern}$$

$$= 6x^2 + 38x + 60 \quad \text{Combine like terms.}$$

b. You can evaluate the polynomial expression $6x^2 + 38x + 60$ by substituting x-values. For instance, to find the total area of the window when $x = 10$, substitute 10 for x.

$$A = 6x^2 + 38x + 60$$

$$= 6(10)^2 + 38(10) + 60$$

$$= 600 + 380 + 60$$

$$= 1040$$

▶ The areas for all five x-values are listed in the table.

x (in.)	10	11	12	13	14
A (in.^{2})	1040	1204	1380	1568	1768

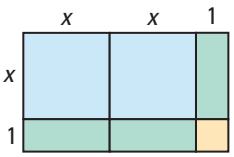
GUIDED PRACTICE

Vocabulary Check ✓

1. How do the letters in "FOIL" help you remember how to multiply two binomials?

Concept Check ✓

2. Write an equation that represents the product of two binomials as shown in the area model at the right.



Explain how to use the distributive property to find the product.

3. $(2x - 3)(x + 4)$

4. $(x + 1)(x^2 - x + 1)$

Skill Check ✓

Use the given method to find the product $(2x + 3)(4x + 1)$.

5. Use an area model.

6. Use the distributive property.

7. Use the FOIL pattern.

8. Which of the methods you used in Exercises 5–7 do you prefer? Explain.

Find the product.

9. $(4x + 7)(-2x)$

10. $-4x^2(3x^2 + 2x - 6)$

11. $(-y - 2)(y + 8)$

12. $(w - 3)(2w + 5)$

13. $(x + 6)(x + 9)$

14. $(-2x - 4)(8x + 3)$

15. $(b + 8)(6 - 2b)$

16. $(-4y + 5)(-7 - 3y)$

17. $(3w^2 - 9)(5w - 1)$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 806.

MULTIPLYING EXPRESSIONS **Find the product.**

18. $(2x - 5)(-4x)$

19. $3t^2(7t - t^3 - 3)$

20. $2x(x^2 - 8x + 1)$

21. $(-y)(6y^2 + 5y)$

22. $4w^2(3w^3 - 2w^2 - w)$

23. $-b^2(6b^3 - 16b + 11)$

DISTRIBUTIVE PROPERTY **Use the distributive property to find the product.**

24. $(t + 8)(t + 5)$

25. $(2d + 3)(3d + 1)$

26. $(4y^2 + y - 7)(2y - 1)$

27. $(3s^2 - s - 1)(s + 2)$

28. $(a^2 + 8)(a^2 - a - 3)$

29. $(x + 6)(x^2 - 6x - 2)$

USING THE FOIL PATTERN **Use the FOIL pattern to find the product.**

30. $(4q - 1)(3q + 8)$

31. $(2z + 7)(3z + 2)$

32. $(x + 6)(x - 6)$

33. $(2w - 5)(w + 5)$

34. $(x - 9)(2x + 15)$

35. $(5t - 3)(2t + 3)$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 18–29
Example 2: Exs. 30–35
Example 3: Exs. 36–47
Example 4: Exs. 36–47
Example 5: Exs. 52–55

MULTIPLYING EXPRESSIONS **Find the product.**

36. $(d - 5)(d + 3)$

37. $(4x + 1)(x - 8)$

38. $(3b - 1)(b - 9)$

39. $(9w + 8)(11w - 10)$

40. $(11t - 30)(5t - 21)$

41. $(9.4y - 5.1)(7.3y - 12.2)$

42. $(3x + 4)\left(\frac{2}{3}x + 1\right)$

43. $\left(n + \frac{6}{5}\right)(4n - 10)$

44. $\left(x + \frac{1}{8}\right)\left(x - \frac{9}{8}\right)$

45. $(2.5z - 6.1)(z + 4.3)$

46. $(t^2 + 6t - 8)(t - 6)$

47. $(-4s^2 + s - 1)(s + 4)$

USING A GRAPH TO CHECK **Find the product. Check your result by comparing a graph of the given expression with a graph of the product.**

48. $(x + 5)(x + 4)$

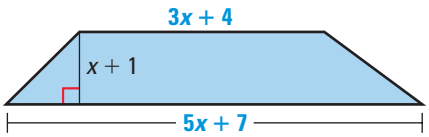
49. $(2x + 1)(x - 6)$

50. $(3x^2 - 8x - 1)(9x + 4)$

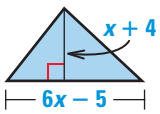
51. $(x + 7)(-x^2 - 6x + 2)$

GEOMETRY CONNECTION **Find an expression for the area of the figure. (*Hint:* the *Table of Formulas* is on p. 813.) Give your answer as a quadratic polynomial.**

52.



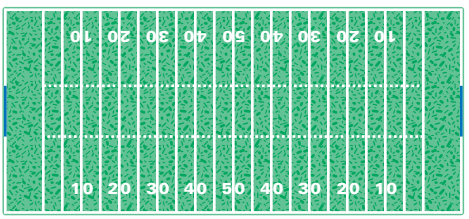
53.



FOOTBALL **In Exercises 54 and 55, a football field's dimensions can be represented by a width of $\left(\frac{1}{2}x + 10\right)$ feet and a length of $\left(\frac{5}{4}x - 15\right)$ feet.**

54. Find an expression for the area A of a football field. Give your answer as a quadratic trinomial.

55. An actual football field is 160 feet wide and 360 feet long. For what value of x does the expression you wrote in Exercise 54 give these dimensions?



VIDEOCASSETTES **In Exercises 56 and 57, use the following information about videocassette sales from 1987 to 1996, where t is the number of years since 1987.**

The annual number of blank videocassettes B sold in the United States can be modeled by $B = 15t + 281$, where B is measured in millions. The wholesale price P for a videocassette can be modeled by $P = -0.21t + 3.52$, where P is measured in dollars. ▶ Source: EIA Market Research Department

56. Find a model for the revenue from annual sales of blank videocassettes. Give the model as a quadratic trinomial.

57. Describe what happens to revenue during the period from 1987 to 1996.

STUDENT HELP

APPLICATION LINK

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BOX OFFICE **In Exercises 58 and 59, use the following information about movie theater admissions from 1980 to 1996, where t is the number of years since 1980.**

The annual number of admissions D into movie theaters can be modeled by $D = 13.75t + 1057.36$, where D is measured in millions. The admission price P can be modeled by $P = 0.11t + 2.90$, where P is the price per person.

▶ Source: Motion Picture Association of America

58. Find a model for the annual box office revenue. Give the model as a quadratic trinomial.

59. What conclusions can you make from your model?

Test Preparation

60. MULTI-STEP PROBLEM Use the table, which gives annual data on cows and milk production (in pounds) in the United States from 1985 to 1996, where t is the number of years since 1985.

Year (t)	0	5	6	7	8	9	10	11
Milk production (1000s of lb/cow)	13	14.8	15	15.6	15.7	16.2	16.4	16.5
Millions of cows	11	10	9.8	9.7	9.6	9.5	9.5	9.4

INTERNET **DATA UPDATE** of U.S. Department of Agriculture data at www.mcdougallittell.com

a. Make a scatter plot of the milk production data.

b. Make a scatter plot of the annual data on the number of cows.

c. Write a linear model for each set of data. (Use the line-fitting technique in Lesson 5.4.)

d. Find a model for the total amount of milk produced. Check to see whether the model matches the information in the table below.

Year (t)	0	5	6	7	8	9	10	11
U.S. milk production (billion lb)	143	148	148	151	151	154	155	154

▶ Source: U.S. Department of Agriculture

e. What conclusions can you make based on the information?

Challenge

61. a. PATTERNS Find each product: $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$. Find a pattern in the results.

b. Use the pattern to predict the product $(x - 1)(x^4 + x^3 + x^2 + x + 1)$. Verify your guess by multiplying or graphing.

EXTRA CHALLENGE
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MIXED REVIEW

SIMPLIFYING EXPRESSIONS **Simplify.** (Review 8.1 for 10.3)

62. $(7x)^2$ **63.** $\left(\frac{1}{3}m\right)^2$ **64.** $\left(\frac{2}{5}y\right)^2$ **65.** $(0.5w)^2$

66. $9^3 \cdot 9^5$ **67.** $(4^2)^4$ **68.** $b^2 \cdot b^5$ **69.** $(4c^2)^4$

70. $(2t)^4 \cdot 3^3$ **71.** $(-w^4)^3$ **72.** $(-3xy)^3(2y)^2$ **73.** $(8x^2y^8)^3$

FINDING SOLUTIONS **Tell how many solutions the equation has.** (Review 9.6)

74. $2x^2 - 3x - 1 = 0$ **75.** $4x^2 + 4x + 1 = 0$ **76.** $3x^2 - 7x + 5 = 0$

77. $7x^2 - 8x - 6 = 0$ **78.** $10x^2 - 13x - 9 = 0$ **79.** $6x^2 - 12x - 6 = 0$

SKETCHING GRAPHS **Sketch the graph of the inequality.** (Review 9.7)

80. $y \geq 4x^2 - 7x$ **81.** $y < x^2 - 3x - 10$ **82.** $y > -2x^2 + 4x + 16$

83. $y < 6x^2 - 1$ **84.** $y \geq x^2 - 3x + 1$ **85.** $y \leq 8x^2 - 3$