10.1 Adding and Subtracting Polynomials

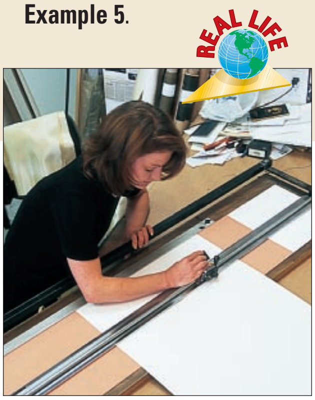
What you should learn

GOAL 1 Add and subtract polynomials.

GOAL 2 Use polynomials to model **real-life** situations, such as energy use in **Exs. 67–69**.

Why you should learn it

▼ To represent **real-life** situations, like mounting a photo on a mat in **Example 5**.



GOAL 1 ADDING AND SUBTRACTING POLYNOMIALS

An expression which is the sum of terms of the form ax^k where k is a nonnegative integer is a **polynomial**. Polynomials are usually written in **standard form**, which means that the terms are placed in descending order, from largest degree to smallest degree.

Polynomial in standard form: $\underbrace{2}_{\text{Leading coefficient}}x^{\overbrace{3}^{\text{Degree}}} + 5x^2 - 4x + \underbrace{7}_{\text{Constant term}}$

The **degree** of each term of a polynomial is the exponent of the variable. The **degree of a polynomial** is the largest degree of its terms. When a polynomial is written in standard form, the coefficient of the first term is the **leading coefficient**.

EXAMPLE 1 *Identifying Polynomial Coefficients*

Identify the coefficients of $-4x^2 + x^3 + 3$.

SOLUTION First write the polynomial in standard form. Account for each degree, even if you must use a zero coefficient.

$$-4x^2 + x^3 + 3 = (1)x^3 + (-4)x^2 + (0)x + 3$$

▶ The coefficients are 1, −4, 0, and 3.

. . . .

A polynomial with only one term is called a **monomial**. A polynomial with two terms is called a **binomial**. A polynomial with three terms is called a **trinomial**.

EXAMPLE 2 *Classifying Polynomials*

	POLYNOMIAL	DEGREE	CLASSIFIED BY DEGREE	CLASSIFIED BY NUMBER OF TERMS
a.	6	0	constant	monomial
b.	$-2x$	1	linear	monomial
c.	$3x + 1$	1	linear	binomial
d.	$-x^2 + 2x - 5$	2	quadratic	trinomial
e.	$4x^3 - 8x$	3	cubic	binomial
f.	$2x^4 - 7x^3 - 5x + 1$	4	quartic	polynomial

To add or subtract two polynomials, add or subtract the like terms. You can use a vertical format or a horizontal format.

STUDENT HELP

INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 3 *Adding Polynomials*

Find the sum. Write the answer in standard form.

a. $(5x^3 - x + 2x^2 + 7) + (3x^2 + 7 - 4x) + (4x^2 - 8 - x^3)$

b. $(2x^2 + x - 5) + (x + x^2 + 6)$

SOLUTION

a. Vertical format: Write each expression in standard form. Align like terms.

$$\begin{array}{rrrr} 5x^3 & + 2x^2 & - x & + 7 \\ & 3x^2 & - 4x & + 7 \\ -x^3 & + 4x^2 & & - 8 \\ \hline 4x^3 & + 9x^2 & - 5x & + 6 \end{array}$$

b. Horizontal format: Add like terms.

$$(2x^2 + x - 5) + (x + x^2 + 6) = (2x^2 + x^2) + (x + x) + (-5 + 6)$$
$$= 3x^2 + 2x + 1$$

EXAMPLE 4 *Subtracting Polynomials*

Find the difference.

a. $(-2x^3 + 5x^2 - x + 8) - (-2x^3 + 3x - 4)$

b. $(x^2 - 8) - (7x + 4x^2)$

c. $(3x^2 - 5x + 3) - (2x^2 - x - 4)$

SOLUTION

a. Use a vertical format. To subtract, you *add the opposite*. This means that you can multiply each term in the subtracted polynomial by -1 and add.

$$\begin{array}{rr} & (-2x^3 + 5x^2 - x + 8) \\ - & (-2x^3 + 3x - 4) \\ \hline \end{array} \quad \text{Add the opposite.} \quad \begin{array}{rrrrr} & -2x^3 & + 5x^2 & - x & + 8 \\ + & 2x^3 & & - 3x & + 4 \\ \hline & & 5x^2 & - 4x & + 12 \end{array}$$

b.

$$\begin{array}{rr} & (x^2 - 8) \\ - & (7x + 4x^2) \\ \hline \end{array} \quad \text{Add the opposite.} \quad \begin{array}{rrrr} & x^2 & & - 8 \\ + & -4x^2 & - 7x & \\ \hline & -3x^2 & - 7x & - 8 \end{array}$$

c. Use a horizontal format.

$$(3x^2 - 5x + 3) - (2x^2 - x - 4) = 3x^2 - 5x + 3 - 2x^2 + x + 4$$
$$= (3x^2 - 2x^2) + (-5x + x) + (3 + 4)$$
$$= x^2 - 4x + 7$$

STUDENT HELP

Study Tip

A common mistake in algebra is to forget to change signs correctly when subtracting one expression from another.

$\cancel{(x^2 - 3x) - (2x - 5x + 4)}$

$\cancel{= x^2 - 3x - 2x - 5x + 4}$

Wrong signs

GOAL 2 USING POLYNOMIALS IN REAL LIFE

REAL LIFE Geometry

EXAMPLE 5 *Subtracting Polynomials*

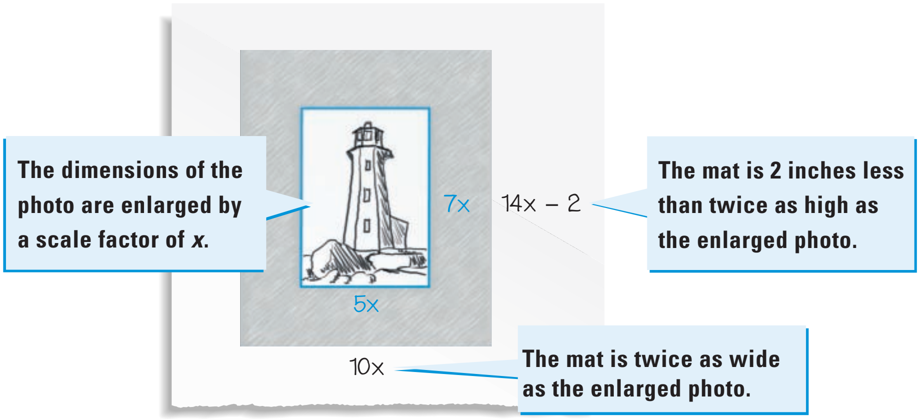
You are enlarging a 5-inch by 7-inch photo by a scale factor of x and mounting it on a mat. You want the mat to be twice as wide as the enlarged photo and 2 inches less than twice as high as the enlarged photo.

a. Draw a diagram to represent the described situation. Label the dimensions.

b. Write a model for the area of the mat around the photograph as a function of the scale factor.

SOLUTION

a. Use rectangles to represent the mat and the photo. Use the description of the problem to label the dimensions as shown in the sample diagram below.



b. Use a verbal model. Use the diagram to find expressions for the labels.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

Area of mat = Total area − Area of photo

LABELS

Area of mat $= A$ (square inches)

Total area $= (10x)(14x - 2)$ (square inches)

Area of photo $= (5x)(7x)$ (square inches)

ALGEBRAIC MODEL

$$A = (10x)(14x - 2) - (5x)(7x)$$
$$= 140x^2 - 20x - 35x^2$$
$$= 105x^2 - 20x$$

▶ A model for the area of the mat around the photograph as a function of the scale factor x is $A = 105x^2 - 20x$.

REAL LIFE Internet

EXAMPLE 6 *Adding Polynomials*

From 1991 through 1998, the number of commercial C and education E Internet Web sites can be modeled by the following equations, where t is the number of years since 1991. ▶ Source: Network Wizards

Commercial sites (in millions): $C = 0.321t^2 - 1.036t + 0.698$

Education sites (in millions): $E = 0.099t^2 - 0.120t + 0.295$

Find a model for the total number S of commercial and education sites.

SOLUTION

You can find a model for S by adding the models for C and E.

$$\begin{array}{r} 0.321t^2 - 1.036t + 0.698 \\ +\ \ 0.099t^2 - 0.120t + 0.295 \\ \hline 0.42t^2 - 1.156t + 0.993 \end{array}$$

▶ The model for the sum is $S = 0.42t^2 - 1.156t + 0.993$.

GUIDED PRACTICE

Vocabulary Check ✓

1. Is $9x^2 + 8x - 4x^3 + 3$ a polynomial with a degree of 2? Explain.

Concept Check ✓

In Exercises 2–4, consider the polynomial expression $5x + 6 - 3x^3 - 4x^2$.

2. Write the expression in standard form and name its terms.

3. Name the coefficients of the terms. Which is the leading coefficient?

4. What is the degree of the polynomial?

ERROR ANALYSIS **Describe the error shown.**

5.

$$\begin{array}{r} 7x^3 - 3x^2 + 5 \\ +\ \ 2x^3 - 5x - 7 \\ \hline 9x^3 - 8x^2 - 2 \end{array}$$

6.

$$\begin{aligned} &(4x^2 - 9x) - (-8x^2 + 3x - 7) \\ &= (4x^2 + 8x^2) + (-9x + 3x) - 7 \\ &= 12x^2 - 6x - 7 \end{aligned}$$

Skill Check ✓

Classify the polynomial by degree and by the number of terms.

7. $-9y + 5$

8. $12x^2 + 7x$

9. $4w^3 - 8w + 9$

10. $\frac{1}{2}x - \frac{3}{4}x^2$

11. -4.3

12. $7y + 2y^3 - y^2 + 3y^4$

Find the sum or the difference.

13. $(x^2 - 4x + 3) + (3x^2 - 3x - 5)$

14. $(-x^2 + 3x - 4) - (2x^2 + x - 1)$

15. $(-3x^2 + x + 8) - (x^2 - 8x + 4)$

16. $(5x^2 - 2x - 1) + (-3x^2 - 6x - 2)$

17. $(4x^2 - 2x - 9) + (x - 7 - 5x^2)$

18. $(2x - 3 + 7x^2) - (3 - 9x^2 - 2x)$

Practice and Applications

Student Help

Extra Practice to help you master skills is on p. 806.

CLASSIFYING POLYNOMIALS Identify the leading coefficient, and classify the polynomial by degree and by number of terms.

19. $-3w + 7$ **20.** $-4x^2 + 2x - 1$ **21.** $8 + 5t^2 - 3t + t^3$

22. $8 + 5y^2 - 3y$ **23.** -6 **24.** $14w^4 + 9w^2$

25. $-\frac{2}{3}x + 5x^4 - \frac{5}{6}$ **26.** $-4.1b^2 + 7.4b^3$ **27.** $-9t^2 + 3t^3 - 4t^4 - 15$

28. $9y^3 - 5y^2 + 4y - 1$ **29.** $-16x^3$ **30.** $-8z^2 + 74 + 39z - 95z^4$

VERTICAL FORMAT Use a vertical format to add or subtract.

31. $(12x^3 + 10) - (18x^3 - 3x^2 + 6)$ **32.** $(a + 3a^2 + 2a^3) - (a^4 - a^3)$

33. $(2m - 8m^2 - 3) + (m^2 + 5m)$ **34.** $(8y^2 + 2) + (5 - 3y^2)$

35. $(3x^2 + 7x - 6) - (3x^2 + 7x)$ **36.** $(4x^2 - 7x + 2) + (-x^2 + x - 2)$

37. $(8y^3 + 4y^2 + 3y - 7) + (2y^2 - 6y + 4)$

38. $(7x^4 - x^2 + 3x) - (x^3 + 6x^2 - 2x + 9)$

HORIZONTAL FORMAT Use a horizontal format to add or subtract.

39. $(x^2 - 7) + (2x^2 + 2)$ **40.** $(-3a^2 + 5) + (-a^2 + 4a - 6)$

41. $(x^3 + x^2 + 1) - x^2$ **42.** $12 - (y^3 + 4)$

43. $(3n^3 + 2n - 7) - (n^3 - n - 2)$ **44.** $(3a^3 - 4a^2 + 3) - (a^3 + 3a^2 - a - 4)$

45. $(6b^4 - 3b^3 - 7b^2 + 9b + 3) + (4b^4 - 6b^2 + 11b - 7)$

46. $(x^3 - 6x) - (2x^3 + 9) - (4x^2 + x^3)$

POLYNOMIAL ADDITION AND SUBTRACTION Use a vertical format or a horizontal format to add or subtract.

47. $(9x^3 + 12) + (16x^3 - 4x + 2)$ **48.** $(-2t^4 + 6t^2 + 5) - (-2t^4 + 5t^2 + 1)$

49. $(3x + 2x^2 - 4) - (x^2 + x - 6)$ **50.** $(u^3 - u) - (u^2 + 5)$

51. $(-7x^2 + 12) - (6 - 4x^2)$ **52.** $(10x^3 + 2x^2 - 11) + (9x^2 + 2x - 1)$

53. $(-9z^3 - 3z) + (13z - 8z^2)$ **54.** $(21t^4 - 3t^2 + 43) - (19t^3 + 33t - 58)$

55. $(6t^2 - 19t) - (3 - 2t^2) - (8t^2 - 5)$

56. $(7y^2 + 15y) + (5 - 15y + y^2) + (24 - 17y^2)$

57. $\left(x^4 - \frac{1}{2}x^2\right) + \left(x^3 + \frac{1}{3}x^2\right) + \left(\frac{1}{4}x^2 - 9\right)$

58. $(10w^3 + 20w^2 - 55w + 60) + (-25w^2 + 15w - 10) + (-5w^2 + 10w - 20)$

59. $(9x^4 - x^2 + 7x) + (x^3 - 6x^2 + 2x - 9) - (4x^3 + 3x + 8)$

60. $(6.2b^4 - 3.1b + 8.5) + (-4.7 + 5.8b^2 - 2.4b^4)$

61. $(-3.8y^3 + 6.9y^2 - y + 6.3) - (-3.1y^3 + 2.9y - 4.1)$

62. $\left(\frac{2}{5}a^4 - 2a + 7\right) - \left(-\frac{3}{10}a^4 + 6a^3\right) - (2a^2 - 7)$

Student Help

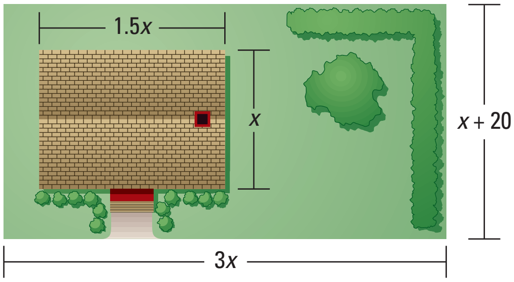
HOMEWORK HELP

Example 1: Exs. 19–30
Example 2: Exs. 19–30
Example 3: Exs. 31–62
Example 4: Exs. 31–62
Example 5: Exs. 63, 64
Example 6: Exs. 65–69

BUILDING A HOUSE **In Exercises 63 and 64, you plan to build a house that is $1\frac{1}{2}$ times as long as it is wide. You want the land around the house to be 20 feet wider than the width of the house, and twice as long as the length of the house, as shown at the right.**

63. Write an expression for the area of the land surrounding the house.

64. If $x = 30$ feet, what is the area of the house? What is the area of the entire property?



POPULATION **In Exercises 65 and 66, use the following information.** Projected from 1950 through 2010, the total population P and the male population M of the United States (in thousands) can be modeled by the following equations, where t is the number of years since 1950.

DATA UPDATE of U.S. Bureau of the Census data at www.mcdougallittell.com

Total population model: $P = 2387.74t + 155{,}211.46$

Male population model: $M = 1164.16t + 75{,}622.43$

65. Find a model that represents the female population F of the United States from 1950 through 2010.

66. For the year 2010, the value of P is 298,475.86 and the value of M is 145,472.03. Use these figures to predict the female population in 2010.

ENERGY USE **In Exercises 67–69, use the following information.** From 1989 through 1993, the amounts (in billions of dollars) spent on natural gas N and electricity E by United States residents can be modeled by the following equations, where t is the number of years since 1989.

▶ Source: U.S. Energy Information Administration

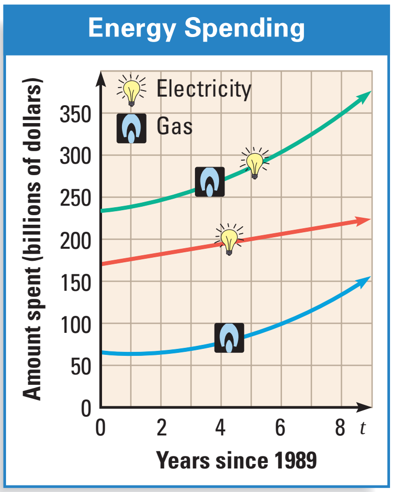
Gas spending model: $N = 1.488t^2 - 3.403t + 65.590$

Electricity spending model: $E = -0.107t^2 + 6.897t + 169.735$

67. Find a model for the total amount A (in billions of dollars) spent on natural gas *and* electricity by United States residents from 1989 through 1993.

68. According to the models, will more money be spent on natural gas or on electricity in 2020?

69. The graph at the right shows U.S. energy spending starting in 1989. Models N, E, and A are shown. Copy the graph and label the models N, E, and A.



FOCUS ON CAREERS

ELECTRICAL ENGINEERS design, test, and monitor the performance of electrical equipment. This includes equipment used by power utilities to generate and transmit electricity.

CAREER LINK
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Test Preparation

70. MULTI-STEP PROBLEM The table below shows the amounts that Megan and Sara plan to deposit in their savings accounts to buy a used car. Their savings accounts have the same annual growth rate g.

Date	1/1/00	1/1/01	1/1/02	1/1/03
Megan	\$250	\$400	\$170	\$625
Sara	\$475	\$50	\$300	\$540

a. On January 1, 2003, the value of Megan's account M can be modeled by $M = 250g^3 + 400g^2 + 170g + 625$, where g is the annual growth rate. Find a model for the value of Sara's account S on January 1, 2003.

b. Find a model for the total value of Megan's and Sara's accounts together on January 1, 2003.

c. The annual growth rate g is equal to $1 + r$, where r is the annual interest rate. The annual interest rate on both accounts is 2.5% for the three-year period. Find the combined value of the two accounts on January 1, 2003.

d. If the used car that Megan and Sara want to buy costs \$2500, will they have enough money?

★ Challenge

71. The sum of any two consecutive integers can be written as $(x) + (x + 1)$. Show that the sum of any two consecutive integers is always odd.

72. Use algebra to show that the sum of any four consecutive integers is always even.

EXTRA CHALLENGE
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MIXED REVIEW

DISTRIBUTIVE PROPERTY **Simplify the expression.** (Review 2.6 for 10.2)

73. $-3(x + 1) - 2$ **74.** $(2x - 1)(2) + x$ **75.** $11x + 3(8 - x)$

76. $(5x - 1)(-3) + 6$ **77.** $-4(1 - x) + 7$ **78.** $-12x - 5(11 - x)$

79. BEST-FITTING LINES Draw a scatter plot. Then draw a line that approximates the data and write an equation of the line. (Review 5.4)

$(-7, 19), (-6, 16), (-5, 12), (-2, 12), (-2, 9), (0, 7),$

$(2, 4), (6, -3), (6, 2), (9, -4), (9, -7), (12, -10)$

EXPONENTIAL EXPRESSIONS **In Exercises 80–85, simplify. Then use a calculator to evaluate the expression. Round the result to two decimal places when appropriate.** (Review 8.1)

80. $(4 \cdot 3^2 \cdot 2^3)^4$ **81.** $(2^4 \cdot 2^4)^2$ **82.** $(-6 \cdot 3^4)^3$

83. $(1.1 \cdot 3.3)^3$ **84.** $5.5^3 \cdot 5.5^4$ **85.** $(2.9^3)^5$

86. ALABAMA The population P of Alabama (in thousands) for 1995 projected through 2025 can be modeled by $P = 4227(1.0104)^t$, where t is the number of years since 1995. Find the ratio of the population in 2025 to the population in 2000. Compare this ratio with the ratio of the population in 2000 to the population in 1995. ▶ Source: U.S. Bureau of the Census (Review 8.3)