1.7

An Introduction to Functions

What you should learn

GOAL 1 Identify a function and make an input-output table for a function.

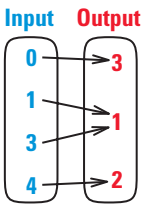
GOAL 2 Write an equation for a **real-life** function, such as the relationship between water pressure and depth in **Example 3**.

Why you should learn it

▼ To represent **real-life** relationships between two quantities such as time and altitude for a rising hot-air balloon in **Example 2**.

GOAL 1 INPUT-OUTPUT TABLES

A **function** is a rule that establishes a relationship between two quantities, called the **input** and the **output**. For each input, there is exactly one output. More than one input can have the same output.



One way to describe a function is to make an **input-output table**. This table lists the inputs and outputs shown in the diagram.

Input	0	1	3	4
Output	3	1	1	2

The collection of all input values is the **domain** of the function and the collection of all output values is the **range** of the function. The table shows that the domain of the function above is 0, 1, 3, 4 and the range is 1, 2, 3.

EXAMPLE 1 *Making an Input-Output Table*

GEOMETRY CONNECTION The diagram shows the first six triangular numbers, 1, 3, 6, 10, 15, 21, which continue on following the same pattern.

Figure 1 Figure 2 Figure 3 Figure 4 Figure 5 Figure 6 and so on.

a. Make an input-output table in which the input is the Figure number n and the output is the corresponding triangular number T.

b. Does the table represent a function? Justify your answer.

c. Describe the domain and the range.

SOLUTION

a. Use the diagram to make an input-output table.

Input n	Output T
1	1
2	3
3	6
4	10
5	15
6	21
…	…

b. Yes, because for each input there is exactly one output.

c. The collection of all input values is the domain: 1, 2, 3, 4, 5, 6, ... The collection of all output values is the range: 1, 3, 6, 10, 15, 21, ...

EXAMPLE 2 *Using a Table to Graph a Function*

You are at an altitude of 250 feet in a hot-air balloon. You turn on the burner and rise at a rate of 20 feet per minute for 5 minutes. Your altitude h after you have risen for t minutes is given by the function

$h = 250 + 20t$, where $0 \leq t \leq 5$.

a. For several inputs t, use the function to calculate an output h.

b. Organize the data into an input-output table.

c. Graph the data in the table. Then use this graph to draw a graph of the function.

d. What is the domain of the function? What part of the domain is shown in the table?

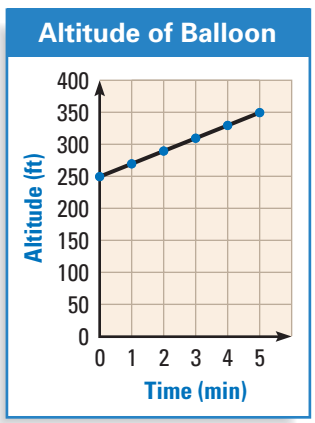
SOLUTION

a. List an output for each of several inputs.

INPUT	FUNCTION	OUTPUT
$t = 0$	$h = 250 + 20(0)$	$h = 250$
$t = 1$	$h = 250 + 20(1)$	$h = 270$
$t = 2$	$h = 250 + 20(2)$	$h = 290$
$t = 3$	$h = 250 + 20(3)$	$h = 310$
$t = 4$	$h = 250 + 20(4)$	$h = 330$
$t = 5$	$h = 250 + 20(5)$	$h = 350$

b. Make an input-output table.

t	0	1	2	3	4	5
h	250	270	290	310	330	350



c. The points in blue are the graph of the table data. The black line is the graph of the function.

d. The domain of the function is all values of t such that $0 \leq t \leq 5$. The values of t shown in the table are 0, 1, 2, 3, 4, 5.

CONCEPT SUMMARY **DESCRIBING A FUNCTION**

Example 1 and Example 2 illustrate that functions can be described in these ways:

- Input-Output Table
- Description in Words
- Equation
- Graph

GOAL 2 WRITING EQUATIONS FOR FUNCTIONS

EXAMPLE 3 *Writing an Equation*

SCUBA DIVING As you dive deeper and deeper into the ocean, the pressure of the water on your body steadily increases. The pressure at the surface of the water is 14.7 pounds per square inch (psi). The pressure increases at a rate of 0.445 psi for each foot you descend. Represent the pressure P as a function of the depth d for every 20 feet you descend until you reach a depth of 120 feet.

a. Write an equation for the function.

b. Create an input-output table for the function.

c. Make a line graph of the function.

SOLUTION

a. To write an equation for the function, apply the problem solving strategy you learned in Lesson 1.5. Write a verbal model, assign labels, and write an algebraic model.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

Pressure at given depth = Pressure at surface + Rate of change in pressure · Diving depth

LABELS

Pressure at given depth = P (psi)

Pressure at surface = **14.7** (psi)

Rate of change in pressure = **0.445** (psi per foot of depth)

Diving depth = d (feet)

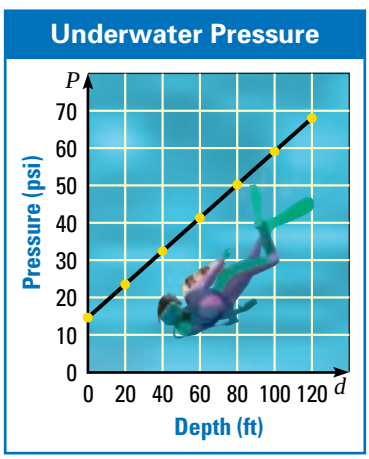
ALGEBRAIC MODEL

$P = 14.7 + 0.445d$ where $0 \leq d \leq 120$

b. Use the equation to make an input-output table for the function. For inputs, use the depths for every 20 feet you descend down to 120 feet.

Depth d	Pressure P
0	14.7
20	23.6
40	32.5
60	41.4
80	50.3
100	59.2
120	68.1

c. You can represent the data in the table with a line graph.



FOCUS ON CAREERS

REAL LIFE **SCUBA DIVING INSTRUCTOR**

Physics and physiology are very important to scuba diving instructors teaching diver safety.

INTERNET CAREER LINK
www.mcdougallittell.com

GUIDED PRACTICE

Vocabulary Check ✓

Complete the sentence.

1. A function is a relationship between two quantities, called the ? and the ?.
2. The collection of all input values is the ? of the function. The collection of all output values is the ? of the function.

Concept Check ✓

3. Four ways to represent a function are (1) ?, (2) ?, (3) ?, and (4) ?.

For a relationship to be a function, it must be true that for each input, there is exactly one output. Does the table represent a function? Explain.

4.

Input	Output
1	3
2	4
3	5
4	6

5.

Input	Output
1	3
2	3
3	4
4	4

6.

Input	Output
1	3
1	4
2	5
3	6

Skill Check ✓

CAR RACING **The fastest winning speed in the Daytona 500 is about 178 miles per hour. Calculate the distance traveled *d* (in miles) after time *t* (in hours) using the equation $d = 178t$.** ▶ Source: NASCAR

7. Copy and complete the input-output table.

Time (hours)	0.25	0.50	0.75	1.00	1.25	1.50
Distance traveled (miles)	?	?	?	?	?	?

8. Describe the domain and the range of the function whose values are shown in the table.
9. Graph the data in the table. Use this graph and the fact that when $t = 0$, $d = 0$ and when $t = 2.81$, $d = 500$ to graph the function.

PRACTICE AND APPLICATIONS

STUDENT HELP

▶ **Extra Practice** to help you master skills is on p. 797.

CRITICAL THINKING **Does the table represent a function? Explain.**

10.

Input	Output
5	3
6	4
7	5
8	6

11.

Input	Output
1	3
2	6
3	11
4	18

12.

Input	Output
9	5
9	4
8	3
7	2

STUDENT HELP

▶ **HOMEWORK HELP**

Example 1: Exs. 10–25
Example 2: Exs. 26–32
Example 3: Exs. 33–36

INPUT-OUTPUT TABLES **Make an input-output table for the function. Use 0, 1, 2, and 3 as the domain.**

13. $y = 3x + 2$
14. $y = 21 - 2x$
15. $y = 5x$
16. $y = 6x + 1$
17. $y = 2x + 1$
18. $y = x + 4$

MAKING INPUT-OUTPUT TABLES **Make an input-output table for the function. Use 1, 1.5, 3, 4.5, and 6 as the domain.**

19. $y = 4x + 2.5$ **20.** $y = 32 - 3x$ **21.** $y = \frac{9}{x} + 10$

22. $y = 2 + \frac{x}{0.5}$ **23.** $y = x^2 - 0.5$ **24.** $y = 1.5 + x^2$

25. **APPLE TREES** A large apple tree may absorb 360 liters of water from the soil per day. The amount of water W absorbed over a short period of time is modeled by the function $W = 360d$, where d represents the number of days. Copy and complete the table.

Input	Function	Output
$d = 1$	$W = 360 \cdot 1$	$W = 360$
$d = 2$	$W = ?$	$W = ?$
$d = 3$	$W = ?$	$W = ?$
$d = 4$	$W = ?$	$W = ?$
$d = 5$	$W = ?$	$W = ?$

TEMPERATURE **In Exercises 26–29, use the following information.**
You use a Celsius thermometer when measuring temperature in science class. Your teacher asks you to show the relationship between Celsius temperature and Fahrenheit temperature with a graph. Use the equation $F = \frac{9}{5}C + 32$ to convert degrees Celsius C to degrees Fahrenheit F.

Input	Function	Output
$C = 0$	$F = \frac{9}{5}(0) + 32$	$F = 32$
$C = 5$	$F = ?$	$F = ?$
$C = 10$	$F = ?$	$F = ?$
$C = 15$	$F = ?$	$F = ?$
$C = 20$	$F = ?$	$F = ?$

26. Copy and complete the input-output table for the function.

Input C	0	5	10	15	20	25	30	35	40
Output F	32	?	?	?	?	?	?	?	?

27. Graph the data in the table. Label the vertical axis Temperature (°C) and the horizontal axis Temperature (°F). Use this graph to graph the function.

28. *Writing* Explain why $F = \frac{9}{5}C + 32$ represents a function.

29. Describe the domain and the range of the function whose values are shown in the table.

STUDENT HELP
Look Back
For help with drawing line graphs, see p. 42.

DRIVE-IN MOVIE **In Exercises 30–32, use the following information.**
A drive-in movie theater charges \$5 admission per car and \$1.25 admission for each person in the car. The total cost C at the drive-in movie theater is given by $C = 5 + 1.25n$, where n represents the number of people in the car. (Assume a maximum of 7 people per car.)

30. Make an input-output table for the function.

31. Describe the domain and range of the function whose values are shown in the table.

32. Graph the data shown in the input-output table.

STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with problem solving in Exs. 33–36.

ADVERTISING In Exercises 33–36, use the following information.
You are in charge of advertising for the drama club's next performance. You can make two signs from a poster board that is 36 inches long and 24 inches wide. Each poster board costs $.75.

33. Write an equation that shows the relationship between the number of signs and the total cost of the poster board. Let n represent the number of signs and let C represent the total cost of the poster board.

34. Evaluate the equation for $n = 6$, 8, 10, and 12. Organize your results in an input-output table.

35. What do the values in the table represent?

36. Suppose you have $5 to spend on poster board. Do you have enough money to make 14 signs? If not, how much more money do you need?

HOUSESITTING In Exercises 37 and 38, use the following information.
As a summer job, you start a housesitting service in your neighborhood. You agree to get the mail, pick up newspapers, water plants, and feed pets for an initial fee of $5, plus $2 per day.

37. Write an equation that shows the relationship between the number of days you housesit and the amount of money you earn for each house.

38. How much money will you earn housesitting one house for one week?

Test Preparation

MULTI-STEP PROBLEM In Exercises 39–43, use the following information.
You start a portable catering business. One of your specialties is a barbecue sandwich plate that costs $.85 to prepare. Suppose you cater an auction where you sell each sandwich plate for $2.00.

39. Write a function that gives the profit you expect from catering the auction.

40. You must also spend $50 on equipment and supplies to cater the auction. Write a function that includes this cost.

41. Use your equation from Exercise 40 to find the profit you will earn if you sell 75 barbecue sandwich plates.

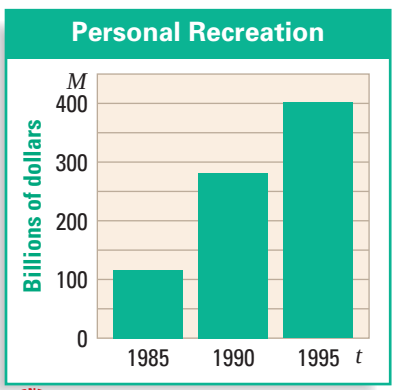
42. Suppose the cost of sandwich rolls increased by $.05 each. What effect do you think this will have on the profit? Write a function that includes this cost.

43. Use your equation from Exercise 42 to find how many barbecue sandwich plates you must sell to make a $100 profit.

Challenge

44. PERSONAL RECREATION
The amount of money spent on personal recreation in billions of dollars M can be modeled by $M = 116.3 + 28.5t$, where t is the number of years since 1985. The bar graph shows the amount of money spent on personal recreation in 1985, 1990, and 1995.

Use a calculator to predict the amount of money spent in billions of dollars in the years 2000 and 2005.



INTERNET DATA UPDATE U.S. Bureau of Economic Analysis at www.mcdougallittell.com

EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

Write the expression in exponential form. (Review 1.2)

45. x raised to the sixth power

46. nine cubed

47. $y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y$

48. $15 \cdot 15 \cdot 15 \cdot 15$

Evaluate the expression. (Review 1.3)

49. $\frac{3b + 3c}{5}$ when $b = 1$ and $c = 9$

50. $6 + \frac{x + 2}{y}$ when $x = 7$ and $y = 3$

51. $3y^2 + w$ when $y = 8$ and $w = 27$

52. $2s^3 - 3t$ when $s = 4$ and $t = 6$

Check whether the number is a solution of the equation or the inequality. (Review 1.4 for 2.1)

53. $7y + 2 = 4y + 8; 2$

54. $5x - 1 = 3x + 2; 4$

55. $32 - 5y > 11; 3$

56. $7m + 2 < 12; 1$

57. $s - 7 \geq 12 - s; 9$

58. $n + 2 \leq 2n - 2; 4$

59. **SHOE SALES** A major athletic footwear company had about \$3750 million in sales of athletic footwear during the year ending May 31, 1997. The company's sales fell to about \$3500 million in 1998. By how much did the company's sales decrease? (Review 1.5)

QUIZ 3

Self-Test for Lessons 1.6 and 1.7

ARTS ACTIVITIES In Exercises 1 and 2, use the table which shows the percent of 18-to-24-year-olds that attended various arts activities at least once in 1997. (Lesson 1.6)

Arts Activities Attended by 18-to-24-year-olds				
Jazz	Musical play	Non-musical play	Art museum	Historic park
15%	26%	20%	38%	46%

▶ Source: Statistical Abstract of the United States, 1998

1. Make a bar graph of the data.

2. What conclusions can you draw from the bar graph?

3. Which equation describes the function that contains all of the data points shown in the table? (Lesson 1.7)

Input x	0	1	2	3	4	5
Output y	1	2	5	10	17	26

A. $y = x^2$ **B.** $y = x^2 + 1$ **C.** $y = 2x + 3$ **D.** $y = x^2 - 1$

HOT-AIR BALLOONING You are at an altitude of 200 feet in a hot-air balloon. You turn on the burner and rise at a rate of 25 feet per minute for 6 minutes. (Lesson 1.7)

4. Write an equation for your altitude as a function of time.

5. Make an input-output table for the function.