1.1

Variables in Algebra

GOAL 1 EVALUATING VARIABLE EXPRESSIONS

A **variable** is a letter that is used to represent one or more numbers. The numbers are the **values** of the variable. A **variable expression** is a collection of numbers, variables, and operations. Here are some examples.

VARIABLE EXPRESSION	MEANING	OPERATION
$8y$ $8 \cdot y$ $8(y)$	8 times y	Multiplication
$\dfrac{16}{b}$ $16 \div b$	16 divided by b	Division
$4 + s$	4 plus s	Addition
$9 - x$	9 minus x	Subtraction

The expression $8y$ is usually not written as $8 \times y$ because of possible confusion with the variable x. Replacing each variable in an expression by a number is called **evaluating the expression**. The resulting number is the value of the expression.

Write the variable expression.	→	**Substitute values for variables.**	→	**Simplify the numerical expression.**

EXAMPLE 1 *Evaluating a Variable Expression*

Evaluate the expression when $y = 2$.

a. $8y$ **b.** $\dfrac{10}{y}$ **c.** $y + 3$ **d.** $14 - y$

SOLUTION

a. $8y = 8(2)$ **Substitute 2 for *y*.**

     $= 16$ **Simplify.**

b. $\dfrac{10}{y} = \dfrac{10}{2}$ **Substitute 2 for *y*.**

     $= 5$ **Simplify.**

c. $y + 3 = 2 + 3$ **Substitute 2 for *y*.**

     $= 5$ **Simplify.**

d. $14 - y = 14 - 2$ **Substitute 2 for *y*.**

     $= 12$ **Simplify.**

EXAMPLE 2 *Evaluating a Real-Life Expression*

Average speed is given by the following formula.

$$\text{Average speed} = \frac{\text{Distance}}{\text{Time}} = \frac{d}{t}$$

Find the average speed (in miles per hour) of a car that traveled 180 miles from Boise, Idaho, to the Minidoka National Wildlife Refuge in 3 hours.
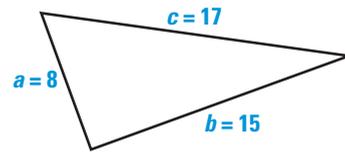
SOLUTION

$\text{Average speed} = \dfrac{d}{t}$	**Write expression.**
$= \dfrac{180}{3}$	**Substitute 180 for *d* and 3 for *t*.**
$= 60$	**Simplify.**

▶ The average speed was 60 miles per hour.

EXAMPLE 3 *Evaluating a Geometric Expression*

GEOMETRY CONNECTION The perimeter of a triangle is equal to the sum of the lengths of its sides: $a + b + c$.

Find the perimeter of the triangle. The dimensions are in feet.

$c = 17$
$a = 8$
$b = 15$

SOLUTION

$\text{Perimeter} = a + b + c$	**Write expression.**
$= 8 + 15 + 17$	**Substitute values.**
$= 40$	**Simplify.**

▶ The triangle has a perimeter of 40 feet.

EXAMPLE 4 *Evaluating Simple Interest*

The simple interest earned by money *P* (called the *principal*) at an annual interest rate *r* for *t* years is given by *Prt*. You deposit $650 at a rate of 8% per year. How much simple interest will you earn after one half of a year?
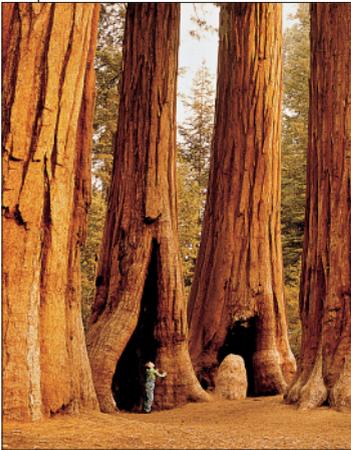
SOLUTION

$\text{Simple interest} = Prt$	**Write expression.**
$= (650)(0.08)(0.5)$	**Substitute 650 for *P*, 0.08 for *r*, and 0.5 for *t*.**
$= 26$	**Simplify.**

▶ After one half of a year, you will have $26 of simple interest.

REAL LIFE **COASTAL REDWOODS,** found in California and Oregon, are the tallest trees on Earth. Only 4% of original redwood forests remains. The tallest living redwood is 370 feet high.
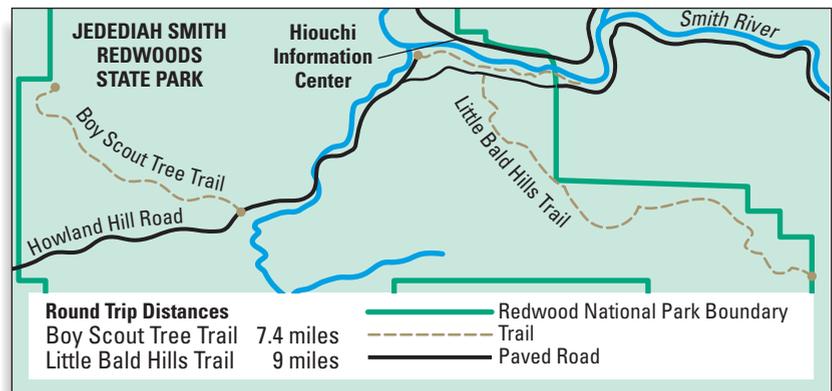
GOAL 2 MODELING A REAL-LIFE SITUATION

Writing the units of each variable in a real-life problem helps you determine the units for the answer. This is called **unit analysis** and it is often used in problem solving in science. When the same units of measure occur in the numerator and the denominator of an expression, you can cancel the units.

In real-life problems, you may need to translate words into mathematics. One way to do this is to use the **verbal model** shown in Example 5.

EXAMPLE 5 *Finding Time*

HIKING AMONG REDWOODS You plan to go hiking in the Jedediah Smith Redwoods State Park in California. You estimate you'll hike at a rate of 2 miles per hour on a steep trail. How long will it take you to hike from Howland Hill Road along the Boy Scout Tree Trail and back?

JEDEDIAH SMITH REDWOODS STATE PARK · Hiouchi Information Center · Smith River · Boy Scout Tree Trail · Little Bald Hills Trail · Howland Hill Road

Round Trip Distances
Boy Scout Tree Trail   7.4 miles
Little Bald Hills Trail   9 miles

——— Redwood National Park Boundary
– – – – Trail
——— Paved Road

▶ Source: Redwood National and State Parks

SOLUTION

The map shows that the round-trip distance is about 7.4 miles.

PROBLEM SOLVING STRATEGY

VERBAL MODEL

$$\text{Time} = \frac{\text{Distance}}{\text{Rate}}$$

LABELS

Time = t (hours)

Distance = **7.4**   (miles)

Rate = **2**   (miles per hour)

ALGEBRAIC MODEL

$t = \dfrac{7.4}{2}$ **Write algebraic model.**

$= 3.7$ **Simplify.**

▶ It should take you about 3.7 hours to hike the Boy Scout Tree Trail.

UNIT ANALYSIS Use unit analysis to check that *hours* are the units of the solution.

$$\text{Time} = \frac{\text{Distance}}{\text{Rate}} = \frac{\text{mi}}{\text{mi/h}} = \text{mi} \cdot \frac{\text{h}}{\text{mi}} = \text{h}$$

GUIDED PRACTICE

1. Explain what it means to *evaluate a variable expression*.

2. What operation is indicated by the expression?

    a. $4y$ **b.** $\dfrac{7}{d}$ **c.** $t + 8$ **d.** $3 - t$

3. Write a variable expression for "5 divided by r."

4. How is unit analysis helpful in solving real-life problems?

Evaluate the expression when $y = 6$.

    5. $5y$ **6.** $\dfrac{24}{y}$ **7.** $y + 19$ **8.** $y - 2$

    9. $y \div 3$ **10.** $27 - y$ **11.** $2.5y$ **12.** $3.2 + y$

Simplify the real-life expression and show unit analysis.

    13. time $= \dfrac{15 \text{ mi}}{5 \text{ mi/h}}$ **14.** perimeter $= 3 \text{ cm} + 4 \text{ cm} + 5 \text{ cm}$

    15. distance $= (60 \text{ mi/h})(2.3 \text{ h})$ **16.** simple interest $= (\$100)\left(\dfrac{0.05}{\text{year}}\right)(2.5 \text{ years})$

🌐 **HIKING** In Exercises 17 and 18, you want to hike a round-trip distance of 10 miles from the Hiouchi Information Center along the Little Bald Hills Trail and back. Calculate how long it will take if you hike at a rate of 1.25 miles per hour.

17. Write a verbal model, provide labels, and write an algebraic model.

18. Show unit analysis.

PRACTICE AND APPLICATIONS

EVALUATING EXPRESSIONS Evaluate the expression for the given value of the variable.

    19. $12x$ when $x = 5$ **20.** $b - 7$ when $b = 24$ **21.** $0.5d$ when $d = 0.5$

    22. $9 + p$ when $p = 11$ **23.** $r(10)$ when $r = 8.2$ **24.** $3.67a$ when $a = 2$

    25. $\dfrac{6.3}{x}$ when $x = 3$ **26.** $\dfrac{2}{3} \cdot x$ when $x = \dfrac{1}{3}$ **27.** $\dfrac{d}{12}$ when $d = 60$

    28. $\dfrac{18}{t}$ when $t = 3$ **29.** $\dfrac{5}{8} - p$ when $p = \dfrac{3}{16}$ **30.** $\dfrac{1}{2} + t$ when $t = \dfrac{1}{2}$

31. 🌐 **INTEREST EARNED** You invest \$80 at a simple annual interest rate of 2%. How much simple interest would you earn in 1.5 years? Use unit analysis to check the units in your response.

32. **GEOMETRY** ▶ **CONNECTION** The perimeter of a square is equal to $4s$, where s is the length of one side. Find the perimeter of the square at the right. Show unit analysis.

7 ft

ERROR ANALYSIS In Exercises 33 and 34, Samantha deposits $300 in an account that earns an annual interest rate of 2.5%. After 9 months, she computes the simple interest.

33. What mistake did Samantha make?

$$(\$300)\left(\frac{2.5}{\text{year}}\right)(0.75\ \text{year}) = \$562.50$$

34. What is the correct amount of simple interest earned after 9 months?

CALCULATING AVERAGE SPEEDS In Exercises 35–38, find the average speed for the given distance and time. Show unit analysis to check units.

35. A train travels 75 miles in 55 minutes.

36. In 5 seconds, an athlete runs 40 feet.

37. A horse gallops 4 kilometers in 30 minutes.

38. Dick Rutan and Jeana Yeager flew nonstop around the world, a distance of 24,986.73 miles, in 216.06 hours. ▶ Source: National Air and Space Museum

39. **DRIVING TIME** If you are driving at a constant speed of 96 kilometers per hour, how long will it take you to travel 288 kilometers?

CALORIES BURNED In Exercises 40–42, use the following information. The number of calories burned while doing an activity can be expressed by rm, where r is the rate of calories burned and m is the number of minutes spent doing the activity.
▶ Source: *Home & Garden* Bulletin No. 72, U.S. Government Printing Office

40. A 120-pound student playing volleyball burns 2.7 calories per minute. If the student plays for 30 minutes, how many calories does the student burn?

41. An in-line skater, who is the same weight as the student in Exercise 40, burns 387 calories in 90 minutes. How many calories does the in-line skater burn per minute?

42. Which activity burns more calories per minute, volleyball or in-line skating?

43. **MOUNTAIN CLIMBING** Heidi Zimmer plans to climb the highest peak in each continent. She has already climbed summits in North America, Europe, Africa, and South America. Copy and complete the table. Convert h, the height in meters, to the height in feet by dividing each given value of h by 0.3048. The last column shows how much shorter each mountain is than Mt. Everest, which is 29,029 feet high.

Mountain, Continent	h	$\frac{h}{0.3048}$	$29{,}029 - \frac{h}{0.3048}$
Mt. McKinley, North America	6194	?	?
Mt. Elbrus, Europe	5633	?	?
Mt. Kilimanjaro, Africa	5963	?	?
Mt. Aconcagua, South America	6959	?	?

▶ Source: Peakware™ World Mountain Encyclopedia

44. 🌎 **BATTING AVERAGE** A baseball player's batting average is found by dividing the number of hits h by the official times at bat b. During the 1998 baseball season, Alex Rodriguez of the Seattle Mariners had 686 official times at bat and made 213 hits. Use a verbal model to find his batting average. Round your answer to the nearest thousandth.

45. MULTIPLE CHOICE You drove 200 miles in 3 hours 20 minutes. Which expression represents your average speed if d represents distance and t represents time?

Ⓐ dt　　　　**Ⓑ** $\dfrac{d}{t}$　　　　**Ⓒ** $\dfrac{t}{d}$　　　　**Ⓓ** $d + t$

46. MULTIPLE CHOICE You invest \$300 at a simple annual interest rate of 4.5%. How much simple interest will you earn in 10 years?

Ⓐ \$115　　　**Ⓑ** \$120　　　**Ⓒ** \$135　　　**Ⓓ** \$150

47. MULTIPLE CHOICE A rectangular computer screen measures 28 centimeters by 21 centimeters. What is the perimeter of the screen?

Ⓐ 49 cm　　　**Ⓑ** 98 cm　　　**Ⓒ** 294 cm　　　**Ⓓ** 588 cm

🌎 **PITCHING SPEED** In baseball the pitcher's mound is 60.5 feet from home plate. The strike zone, or distance across the plate, is 17 inches. The time it takes for a baseball to reach home plate can be determined by dividing the distance the ball travels by the speed at which the pitcher throws the baseball.

48. If a pitcher throws a baseball at 90 miles per hour, how many seconds does it take for the baseball to reach home plate?

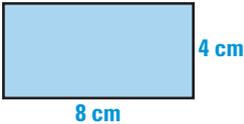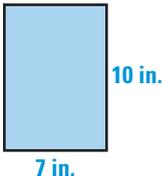**49.** For Exercise 48, how long is the ball in the strike zone?

MIXED REVIEW

FRACTIONS **Perform the indicated operation. Simplify your answer, if possible.** (Skills Review, pp. 781–783)

50. $1\dfrac{7}{8} + \dfrac{3}{4}$　　　**51.** $\dfrac{2}{5} - \dfrac{1}{10}$　　　**52.** $5\dfrac{3}{4} \cdot \dfrac{1}{3}$　　　**53.** $2\dfrac{1}{5} \div \dfrac{4}{5}$

54. $3\dfrac{2}{3} \cdot 3$　　　**55.** $\dfrac{6}{5} \div \dfrac{3}{10}$　　　**56.** $4\dfrac{5}{9} - \dfrac{1}{5}$　　　**57.** $8\dfrac{9}{10} + 1\dfrac{1}{5}$

GEOMETRY **CONNECTION** **Find the area of the rectangle.** (Skills Review, p. 791)

58.

4 cm
8 cm

59.

10 in.
7 in.

60.

6 m
28 m

EXPRESSIONS **Evaluate the expression when $x = 2$.** (Review 1.1 for Lesson 1.2)

61. $2x$　　　**62.** $(x)(x)$　　　**63.** $3x$　　　**64.** $(x)(x)(x)$

65. $4x$　　　**66.** $(x)(x)(x)(x)$　　　**67.** $5x$　　　**68.** $(x)(x)(x)(x)(x)$